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Delta-Isobar Magnetic Form Factor in QCD

V.M. Belyaev*

Institute of Theoretical and Experimental Physics
117259 Moscow, Russia
and
Continuous Electron Beam Accelerator Facility
12000 Jefferson Ave, Newport News, Virginia 23606, USA

We consider the QCD sum rules approach for Δ-isobar magnetic form factor in the infrared region $0 \leq Q^2 < 1GeV^2$. The QCD sum rules in external variable field are used. The obtained form factor is in agreement with quark model predictions for the Δ-isobar magnetic moments.

I. INTRODUCTION

The QCD sum rule method suggested by Shifman, Vainshtein and Zakharov (SVZ) in the pioneering paper [1] becomes now a universal tool for calculating different properties of low-lying hadronic states. Using the original version of this method, the meson [1] and baryon [2] masses were found from the sum rules for two-point correlation functions. Using the three-point correlation functions, hadron form factors at intermediate Q^2 can be obtained [3]. Unfortunately, this method does not work if one tries to calculate form factors in the infrared region $0 < Q^2 < 1GeV^2$ due to power corrections $1/Q^{2n}$ at $Q = 0$. The new method - QCD sum rules in external field was suggested in [4], and using this method nucleon magnetic moments were found [5] as well as baryon axial couplings [6]. Then this method was formulated for a variable external field [7] which gives a possibility to calculate form factors at $Q^2 \neq 0$.

In [7] we have formulated a new method for calculating hadronic form factors in the infrared region. To study a form factor at nonzero Q^2, it is necessary to introduce a variable external field. The calculation of a polarization operator in this field encounters a number of difficulties as compared with the case of a constant external field. The arising problems and methods to avoid them were discussed in detail in the paper [7].

Let us note that in the papers [8] and [9], the pion and nucleon charge radii were considered, using the methods similar to ours. However, the results obtained in [8] are connected not with the calculation of the total form factor $F(Q^2)$, but only the first derivative at zero momentum transfer $< r^2 > \sim F'(Q^2)|_{Q=0}$. In [9] pion form factor was considered. The aim of the paper is to use the general method for calculating Δ-isobar magnetic form factor in the infrared region.

II. POLARIZATION OPERATOR IN VARIABLE EXTERNAL FIELD

To compute Δ-isobar magnetic form factor we shall consider the following correlator in an external variable electromagnetic field:

$$\Pi^{V*}_{\mu\nu}(p,k) = i\int 2\pi\delta(kx)e^{ipx}d^4x < T\{\eta_\mu(x),\bar{\eta}_\nu(0)\} >_V \qquad (2.1)$$

$$= \int_{-\infty}^{+\infty} dz i \int e^{i(p+kz)x} d^4x < T\{\eta_\mu(x),\bar{\eta}_\nu(0)\} >_V$$

$$= \int_{-\infty}^{+\infty} dz \Pi^V_{\mu\nu}(p+kz,k)$$

where

$$\Pi^V_{\mu\nu}(p,k) = i\int e^{ipx}d^4x < T\{\eta_\mu(x),\bar{\eta}_\nu(0)\} >_V \qquad (2.2)$$

$$\eta_\mu(x) = \epsilon^{abc}(u^aC\gamma_\mu u^b)u^c \qquad (2.3)$$

is the quark current with the Δ-isobar quantum numbers suggested in the first paper in Ref.[2], u is the u-quark operator; a , b and c are the color indeces, ϵ^{abc} is the antisymmetric tensor and $C = -C^T$ is the charge conjugation matrix; index V means the vacuum average in the presence of weak external electromagnetic field that is responsible for adding to Lagrangian of the following term

$$\Delta\mathcal{L} = -V_\mu e^{ikx}\sum_f e_f\bar{q}_f(x)\gamma_\mu q_f(x) = -V_\mu j_\mu(x)e^{ikx} \qquad (2.4)$$

where e_f is the charge of the quark with flavor f, V_μ and k_μ are the amplitude and the momentum of the classical external field. This correlator was suggested in [7].

Now let us discuss the reason why we need to introduce the δ-function in the correlator (2.1). To calculate the correlator (2.2) at $p^2 \sim -1GeV^2$, $(p+q)^2 \sim -1GeV^2$, $k^2 = -Q^2 < 0$) we use operator product expansion in the presence of external variable field (2.4). So we need to know nonperturbative quark propagator in the field

$$<: T\{q^a_\alpha(x),q^b_\beta(0)\} :>_V \qquad (2.5)$$

$$=< T\{q^a_\alpha(x),q^b_\beta(0)\} >_V - < T\{q^a_\alpha(x),q^b_\beta(0)\} >_V^{(pert.)}$$

Where :: denotes a subtraction of perturbative contribution. It is possible to find perturbative part of this propagator in the form of expansion over the coupling constant. To take into account nonperturbative interaction of the quark with the external field, we expand eq.(2.5) over x_μ

$$<: T\{q^a_\alpha(x),q^b_\beta(0)\} :>_V =<: q^a_\alpha(0),q^b_\beta(0) :>_V \qquad (2.6)$$

$$+x_\mu <: D_\mu q^a_\alpha(0),q^b_\beta(0) :>_V +\frac{1}{2}x_\mu x_\nu <: D_\mu D_\nu q^a_\alpha(0),q^b_\beta(0) :>_V + ...$$

It is clear that the n'th term of expansion (2.6) can give dimensionless factor $(kx)^n$. Effectively it means that highest terms contribution of expansion (2.6) into a polarization operator (2.2) is not suppressed because this factor $(kx)^n$ corresponds to the factor $(kp)/p^2 \sim 1$. To kill the dangerous contributions of the terms $\sim (kx)^n$ we insert $\delta(kx)$ into the correlator (2.1). This correlator (2.1) can be calculated in a form of series over $1/p^2$. Therefore at respectively large $-p^2$ this correlator can be calculated with a good accuracy using only the first few terms in expansion (2.6).

The nonperturbative quark propagator in the external field has the following form:

$$<: u^a_\alpha(x), \bar{u}^b_\beta(0) :>= e_u \frac{\delta^{ab}}{12}\{\hat{V}_{\alpha\beta} k^2 \Pi_1(k^2)$$

$$+(\sigma_{\rho\lambda})_{\alpha\beta} k_\rho V_\lambda \Pi_2(k^2) + (Vx)[i <: \bar{\psi}\psi :>_0 + \frac{1}{2} k^2 \Pi_2(k^2)]\delta_{\alpha\beta} \qquad (2.7)$$

$$+\frac{i}{2}\hat{V}_{\alpha\beta}(kx)k^2\Pi_1(k^2) + \frac{i}{2}(\sigma_{\rho\lambda})_{\alpha\beta}k_\rho V_\lambda(kx)\Pi_2(k^2)$$

$$-\frac{1}{4}x_\mu \varepsilon_{\mu\nu\rho\lambda}(\gamma_\lambda\gamma_5)_{\alpha\beta}k_\nu V_\rho k^2 \Pi_1(k^2)$$

$$+\frac{i}{12}x^2(\sigma_{\rho\lambda})_{\alpha\beta}k_\rho V_\lambda[<: \bar{\psi}\psi :>_0 - i\Pi^G_1(k^2) - 2\Pi^G_2(k^2) + ik^2\Pi_4(k^2)]$$

$$+(Vx)(kx)k^2\Pi_3(k^2)\delta_{\alpha\beta} + \frac{(kx)^2}{2}(\sigma_{\rho\lambda})_{\alpha\beta}k_\rho V_\lambda \Pi_4(k^2)$$

$$+\frac{(kx)}{12}x_\mu(\sigma_{\mu\nu})_{\alpha\beta}V_\nu[\frac{5}{2}i <: \bar{\psi}\psi :>_0 + \Pi^G_1(k^2) + 3ik^2\Pi_3(k^2) + i\Pi^G_2(k^2)$$

$$-\frac{5}{2}k^2\Pi_4(k^2)] + \frac{(Vx)}{12}x_\mu(\sigma_{\mu\nu})_{\alpha\beta}k_\nu[\frac{i}{2} <: \bar{\psi}\psi :>_0 - \Pi^G_1(k^2)$$

$$-i\Pi^G_2(k^2) + 3ik^2\Pi_3(k^2) - \frac{k^2}{2}\Pi_4(k^2)]$$

$$+ \,(terms\ with\ an\ odd\ number\ of\ \gamma - matrices)\} + O(x^3)$$

The correlators $\Pi_i(k^2)$ are defined as follows:

$$\begin{aligned}
&e_u\Pi_1(k^2)(k^2 g_{\mu\nu} - k_\mu k_\nu) = \\
&i\int e^{ikx} d^4x <: T\{\sum_f \bar{q}_f \gamma_\mu q_f(x), \bar{u}\gamma_\nu u(0)\} :>_0 \\
&e_u\Pi_1(k^2)(k_\mu g_{\nu\rho} - k_\nu g_{\mu\rho}) = \\
&i\int e^{ikx} d^4x <: T\{\sum_f \bar{q}_f \gamma_\rho q_f(x), \bar{u}\sigma_{\mu\nu} u(0)\} :>_0 \\
&e_u\Pi_3(k^2)(k^2(g_{\mu\rho}k_\nu + g_{\nu\rho}k_\mu) - 2k_\mu k_\nu k_\rho) = \\
&i\int e^{ikx} d^4x <: T\{\sum_f e_f \bar{q}_f \gamma_\rho q_f(x), \bar{u}\ D_{\{\mu}\overrightarrow{D}_{\nu\}}\ u(0)\} :>_0 \\
&e_u\Pi_4(k^2)k_\mu k_\nu(g_{\rho r}k_p - g_{\rho p}k_r) + \ldots = \\
&i\int e^{ikx} d^4x <: T\{\sum_f e_f \bar{q}_f \gamma_\rho q_f(x), \bar{u}\sigma_{pr}\ D_{\{\mu}\overrightarrow{D}_{\nu\}}\ u(0)\} :>_0 \\
&e_u\Pi^G_1(k^2)(k_\mu g_{\rho\nu} - k_\nu g_{\rho\mu}) = \\
&i\int e^{ikx} d^4x <: T\{\sum_f e_f \bar{q}_f \gamma_\rho q_f(x), g_s\bar{u}G^n_{\mu\nu}t^n u(0)\} :>_0 \\
&e_u\Pi^G_2(k^2)(k_\mu g_{\rho\nu} - k_\nu g_{\rho\mu}) = \\
&i\int e^{ikx} d^4x <: T\{\sum_f e_f \bar{q}_f \gamma_\rho q_f(x), g_s\bar{u}\tilde{G}^n_{\mu\nu}t^n\gamma_5 u(0)\} :>_0
\end{aligned} \qquad (2.8)$$

Perturbative contributions are subtracted in correlators (2.8). These expressions were obtained in [7].

In this paper we neglect operators $\Pi^G(k^2)$ because their contribution into a sum rule is small (see [7]).

III. THE SUM RULES

In this Section, we obtain a sum rule for the Δ-isobar magnetic form factor. First, we should choose a tensor structure which has a contribution from the magnetic transition between baryon states with quantum numbers $J = 3/2$. To this end, we consider the contribution of two baryons with masses m_1 and m_2 into the polarization operator $\Pi^V(p,k)$ (2.2)

$$\frac{V_\rho < 0 \mid \eta_\mu \mid \Delta_1 >< \Delta_1 \mid j^{em}_\rho \mid \Delta_2 >< \Delta_2 \mid \bar{\eta}_\nu \mid 0 >}{(p^2 - m_1^2)((p+k)^2 - m_2^2)}, \qquad (3.1)$$

where Δ_1 and Δ_2 are baryon states with masses m_1 and m_2 respectively. Here we consider the case when only spinor parts of the Rarita-Shwinger fields interact with a photon. In such case, the matrix element of the electromagnetic current has the following form:

$$< N_1 \mid j_\rho^{em} \mid N_2 >=$$
$$\bar{v}_\mu^{(1)}(p) g_{\mu\nu}[f_{12}(k^2)\gamma_\rho + \frac{\varphi_{12}(k^2)}{m_1+m_2}\sigma_{\rho\lambda}k_\lambda + \psi_{12}(k^2)k_\rho]v_\nu^{(2)}(p+k) =$$
$$\bar{v}_\mu^{(1)}(p) g_{\mu\nu}[(f_{12}(k^2)+\varphi_{12}(k^2))\gamma_\rho + \mathcal{P}_\rho\frac{\varphi_{12}(k^2)}{m_1+m_2} + \psi_{12}(k^2)k_\rho]v_\nu^{(2)}(p+k) \quad (3.2)$$
$$< 0 \mid \eta_\mu \mid N, J^P = 3/2^+ >= \lambda v_\mu(p)$$
$$\mathcal{P}_\mu = p_\mu + (p+k)_\mu,$$

where $v_\mu(p)$ is a Rarita-Shwinger spin-vector satisfying the Dirac equation: $(\hat{p} - m)v_\mu(p) = 0$, $\gamma_\mu v_\mu = 0$, $p_\mu v_\mu = 0$.

Using (3.2) we can transform (3.1) to the following form:

$$\frac{\lambda_1\lambda_2}{(p^2-m_1^2)((p+k)^2-m_2^2)}V_\mu[g_{\mu\nu}\hat{p}_1\gamma_\rho\hat{p}_2 G_M(k^2)/3 \quad (3.3)$$

$+(other\ structures\ with\ \gamma_\mu\ placed\ at\ the\ beginning\ and\ \gamma_\nu\ at\ the\ end\ of\ them)]$

where G_M is the magnetic form factor.

It is important to note that there is no spin-1/2 baryon contribution in the structure $g_{\mu\nu}\hat{p}_1\gamma_\rho\hat{p}_2$ which has the following amplitude:

$$< 0 \mid \eta_\rho \mid J = 1/2 >= (Ap_\mu + B\gamma_\mu)u(p)$$

where $(\hat{p} - m)u(p) = 0$ and $Am + 4B = 0$.

From (3.3), it is obvious that the structure $g_{\mu\nu}\hat{p}_1\gamma_\rho\hat{p}_2$ (where $p_1 = p$ and $p_2 = p + k$) contains magnetic transition only, since $G_M^{12}/3 = f_{12}(k^2) + \varphi_{12}(k^2)$, where G_M^{12} is the magnetic form factor. So, we shall further consider only the structure $g_{\mu\nu}\hat{p}_1\gamma_\rho\hat{p}_2$.

Now let us discuss the factor 1/3 which have appeared in eq.(3.3). Consider interaction of spin-3/2 particle with the electromagnetic field:

$$a(\overline{\Psi}_\mu(P_1)g_{\mu\nu}(P_1+P_2)_\rho\Psi_\nu(P_2) + ib\overline{\Psi}_\mu(\sigma_{\rho\lambda}g_{\mu\nu}/2 + 2g_{\mu\rho}g_{\nu\lambda})\Psi_\nu F_{\rho\lambda} \quad (3.4)$$

where $\frac{1}{2}\sigma_{\rho\lambda} = [\gamma_\rho\gamma_\lambda]$, Ψ_μ is Rarita-Shwinger spin-vector field ($(\hat{P} - m) = 0, \gamma_\mu\Psi_\mu = 0)$, $F_{\rho\lambda} = \partial_\rho A_\lambda - \partial_\lambda A_\rho$. The first term of eq.(3.4) corresponds to the spin-independent part of electromagnetic interaction of $\frac{3}{2}$-spin particle

and the second one describes the spin-dependent interaction. To express the value of the magnetic moment (at $Q^2 = 0$) let us consider the case when $A_0 = 0, P_0 = m, F_{\rho\lambda} = \delta_{\rho i}\delta_{\lambda j}\epsilon_{ijk}H_k$, where H_k is magnetic field. Then we have

$$ib\overline{\Psi}_m(g_{mn}\sigma_{ij}\epsilon_{ijk}/2 + 2\epsilon_{mnk})\Psi_n H_k = b\overline{\Psi}_m(g_{mn}\Sigma_k + 2i\epsilon_{mnk})\Psi_n H_k \quad (3.5)$$

where $\Sigma_k = diag(\sigma_k, \sigma_k)$ Now we see that the operator $(\Sigma_k g_{mn} + 2i\epsilon_{mnk})$ is equal to $2S_k$ where S_k is spin operator for the Rarita-Shwinger field. So, the maximal energy of the particle in the magnetic field is equal to

$$E_{int.} = 3bH = \mu H, \quad (3.6)$$

where μ, by definition, is magnetic moment or magnetic form factor at $Q^2 = 0$. Thus, we have

$$\mu = 3b \quad (3.7)$$

Now let us consider the double dispersional relation for the function at tensor structure $g_{\mu\nu}\hat{p}_1\gamma_\rho\hat{p}_2 V_\rho$:

$$f(P_1^2, P_2^2, Q^2) = \frac{1}{\pi^2}\int_0^\infty\int_0^\infty \frac{\rho(s_1, s_2, Q^2)}{(P_1^2+s_1)(P_2^2+s_2)}, \quad (3.8)$$

where $P_1^2 = -p_1^2$, $P_2^2 = -p_2^2$, $Q^2 = -k^2 \geq 0$ and $\rho(s_1, s_2, Q^2)$ is the spectral density. Due to reasons mentioned above, we cannot calculate $\rho(p_1, p_2, Q^2)$ directly, but we can consider the double Borel transformed structure function of correlator (2.1). Notice, that under replacement $p_1 \to p_1 + kz$, $p_2 \to p_2 + kz$ the structure $\hat{p}_1\gamma_\mu\hat{p}_2 V_\mu$ transforms to

$$\hat{p}_1\gamma_\mu\hat{p}_2 V_\mu \to (\hat{p}_1 + \hat{k}z)\hat{V}(\hat{p}_2 + \hat{k}z) = \hat{p}_1\hat{V}\hat{p}_2 - 2(Vp_1)\hat{p}_1 z - \quad (3.9)$$
$$2(Vp_2)\hat{p}_2 z + (p_1^2 + p_2^2)z\hat{V} + z^2(2(Vk)\hat{k} - \hat{V}k^2)$$

and all other structures in (3.3) with smaller number of $\gamma-$matrices near $g_{\mu\nu}$, could not be transformed into $\hat{p}_1\hat{V}\hat{p}_2$. So we can extract $g_{\mu\nu}\hat{p}_1\hat{V}\hat{p}_2$ in the integral (2.1). Then the integral representation for the double Borel transformed structure function under consideration is obtained from (3.8) by applying the operator $\hat{O}$:

$$\hat{O}f(P_1^2, P_2^2) = \int_{-\infty}^{+\infty} dz \hat{B}_{P_1^2}\hat{B}_{P_2^2} f((P_1+kz)^2, (P_2+kz)^2) \quad (3.10)$$

where

$$\hat{\mathcal{B}} = \lim_{n\to\infty} \frac{(P^2)^{n+1}}{n!}\left(-\frac{\partial}{\partial P^2}\right)^n$$

$$P^2/n = M^2$$

M^2 is the Borel parameter.

Applying the operator $\hat{O}$ to the left-hand and right-hand sides of double dispersion relation (3.8) we get

$$f(M^2,Q^2) = \frac{1}{\pi^2}\int_{-1/2}^{+1/2}\exp(\frac{Q^2z^2}{M^2})\times$$
$$\int_0^\infty\int_0^\infty ds_1 ds_2 \exp(-\frac{s_1+s_2}{2M^2}+\frac{(s_1-s_2)z}{M^2})\rho(s_1,s_2,Q^2) \qquad (3.11)$$

where $M_1^2 = M_2^2 = 2M^2$.

We shall use the sum rules (3.11) to calculate the nucleon form factor. The function $f(M^2,Q^2)$ is calculated using operator expansion in the external field, and the spectral density $\rho(s_1,s_2,Q^2)$ is saturated by intermediate states with quantum numbers of the current (2.3).

There are two different types of intermediate state contributions into (3.11). The first is responsible for the diagonal transitions between the states with equal masses. The second is responsible for nondiagonal transitions between the states with different masses. In the first case the right-hand side of (3.11) obviously has the form

$$\lambda^2\frac{G_M}{3}(Q^2)e^{-m^2/M^2}\int_{-1/2}^{+1/2}e^{\frac{Q^2z^2}{M^2}}dz \qquad (3.12)$$

where λ^2 is the square of the residue of the state with mass m into the current η_μ defined by formula (3.2), $G_M(Q^2)$ is the corresponding magnetic form factor.

In the second case for the transition between states with masses m_1 and m_2 we get:

$$\lambda_1\lambda_2\frac{G_M^{(12)}}{3}(Q^2)e^{-\frac{m_1^2+m_2^2}{2M^2}}\int_{-1/2}^{+1/2}e^{\frac{Q^2z^2+(m_1^2-m_2^2)z}{M^2}}dz \qquad (3.13)$$

Now, to investigate the properties of (3.12) and (3.13) let us expand them in the series on Q^2/M^2

$$\lambda^2\frac{G_M(Q^2)}{3}e^{-m^2/M^2}(1+\frac{1}{12}\frac{Q^2}{M^2}+\frac{1}{5\cdot 2^5}(\frac{q^2}{M^2})^2+\dots) \qquad (3.14)$$

$$\beta_1\beta_2 G_M^{12}(Q^2)e^{-\frac{m_1^2+m_2^2}{2M^2}}\frac{2M^2}{m_1^2-m_2^2}\sinh(\frac{m_1^2-m_2^2}{2M^2})\times \qquad (3.15)$$
$$\{1+\frac{Q^2}{M^2}[\frac{1}{4}-\frac{M^2}{m_1^2-m_2^2}\coth(\frac{m_1^2-m_2^2}{2M^2})+\frac{2M^4}{(m_1^2-m_2^2)^2}]+\dots\}$$

From (3.14) and (3.15) we see that diagonal transitions of the excited states are exponentially suppressed compared to the Δ-isobar contribution in (3.14). Let us write the non-suppressed part of the contribution from the nondiagonal transition between the nucleon with mass m_N and an excited state with mass m_{Δ^*}

$$\lambda\lambda_{\Delta^*}\frac{G_M^{\Delta\Delta^*}(Q^2)}{3}\frac{e^{-\frac{m_\Delta^2}{M^2}}M^2}{m_{\Delta^*}^2-m_\Delta^2}\{1+$$
$$\frac{Q^2}{M^2}[\frac{1}{4}-\frac{M^2}{m_{\Delta^*}^2-m_\Delta^2}\coth(\frac{m_{\Delta^*}^2-m_\Delta^2}{2M^2})+\frac{2M^4}{(m_{\Delta^*}^2-m_\Delta^2)^2}]+\dots\} \qquad (3.16)$$

where m_Δ, m_{Δ^*}, λ and λ_{Δ^*} are masses and residues of Δ-isobar and its resonance Δ^* respectively.

Expression (3.16) is analogous to the contribution from the single-pole term appearing in QCD sum rules for correlators in constant external field (see [4]).

It is easy to see that the function multiplied by Q^2/M^2 in (3.16) changes from $1/12$ at $m_2^2-m_\Delta^2\to 0$ to $1/4$ at $m_2^2-m_\Delta^2\to\infty$. However, taking into account the continuum, only contributions from the states with $m_2^2-m_\Delta^2\sim s_0-m_\Delta^2$ (s_0 is the continuum threshold) are to be considered. In the region $s\gg s_0$, our model of continuum is quite correct, but when $s\sim s_0$ it is not so. Then we see, that nonexponentially suppressed terms will be determined by the states with $m_2^2\sim s_0$. Taking $m_2^2-m_\Delta^2\simeq s_0-m_\Delta^2\simeq 1.5GeV^2$, $M^2\sim 1GeV^2$, expression (3.16) can be written in the form

$$\lambda_\Delta\lambda_{\Delta^*}\frac{G_M^{\Delta\Delta^*}(Q^2)}{3}\frac{e^{-\frac{m_\Delta^2}{M^2}}M^2}{m_{\Delta^*}^2-m_\Delta^2}(1+\frac{Q^2}{12M^2}(1+\epsilon)+\dots) \qquad (3.17)$$

where $\epsilon < 0.1$.

Thus, from (3.14) and (3.17) it is seen that when $Q^2/M^2\le 1$, and $M^2\sim 1GeV^2$, the right-hand side of (3.11) is

$$\frac{1}{3}\lambda_\Delta^2 e^{-m_\Delta^2/M^2}(G_M(Q^2)+C(Q^2)M^2)\int_{-1/2}^{+1/2}e^{\frac{Q^2z^2}{M^2}}dz \qquad (3.18)$$

The accuracy of (3.18) is of an order of a few percents (it depends on the numerical value of ϵ from (3.17)). It can be shown, that the next terms in the expansion (3.15) in powers of Q^2/M^2 do not change the situation. So, in the region $Q^2/M^2 \leq 1$, $M^2 \sim 1GeV^2$, the right side of the sum rule (3.11) is indeed represented by expression (3.18).

Let us note that when $Q^2/M^2 > 1$, the expression (3.18) is invalid and we cannot separate the contribution of the single-pole terms from the contribution of the us double-pole term $\sim G_M(Q^2)$ which we are interested in. Thus, our sum rule is expected to be valid in the region $0 \leq Q^2 < 1GeV^2$.

Here we have constructed the right-hand "phenomenological" side of the sum rule (3.11) and discussed the region of its applicability. Now, let us pass to the calculation of the left, "theoretical" part of our sum rule.

Using eq.(2.7) and dispersion integral (3.11) we have obtained the following sum rule for G_M:

$$\int_{-1/2}^{+1/2} e^{Q^2 z^2/M^2} dz \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 e^{-\frac{s_1+s_2}{2M^2}+z\frac{s_1-s_2}{M^2}} \frac{\rho_\Delta(s_1,s_2,Q^2)}{\pi^2}$$

$$+\frac{2}{3}a^2 \int_{-1/2}^{+1/2} dz e^{Q^2 z^2/M^2} \tag{3.19}$$

$$+\frac{2}{3}(2\pi)^2 a(i\Pi_2(Q^2))M^2 e^{\frac{Q^2}{4M^2}} - \frac{2}{9}a^2(1+Q^2(i\Pi_4(Q^2)))e^{\frac{Q^2}{4M^2}}$$

$$-(2\pi)^2\frac{Q^2}{3}\Pi_1(Q^2)e^{\frac{Q^2}{4M^2}} = I(M^2,s_0,Q^2)\int_{-1/2}^{+1/2} e^{\frac{Q^2 z^2}{M^2}} dz$$

$$= \tilde{\lambda}_\Delta^2 e^{\frac{-m_\Delta^2}{M^2}} e_u^{-1}(\frac{1}{3}G_M(Q^2) + C(Q^2)M^2 \int_{-1/2}^{+1/2} e^{\frac{Q^2 z^2}{M^2}} dz$$

where $e_u = +\frac{2}{3}$ and $\rho_\Delta(s_1,s_2,Q^2)$ is a spectral density:

$$\frac{1}{\pi^2}\rho_\Delta(s_1,s_2,Q^2) = Q^2\frac{(\kappa-U)^2}{2^6\kappa^3}\left(\frac{8}{3}(2\kappa+U) + Q^2(1+\frac{U(2\kappa+U)}{\kappa^2})\right)$$

where $\kappa = \sqrt{(s_1+s_2+Q^2)-4s_1s_2}$, $U = s_1+s_2+Q^2$, $a = (2\pi)^2 |< \bar\psi\psi >_0| = 0.55GeV^3$, $\tilde\lambda_\Delta = (2\pi)^2\lambda_\Delta = 2.3GeV^6$, $s_0 = 4GeV^2$, $m_\Delta = 1.232GeV$,

$$i\Pi_2(Q^2) = -< \bar\psi\psi >_0 \left(\frac{4}{Q^2+0.6} - \frac{2}{Q^2+2.6}\right)(GeV)$$

Here Q^2 is in units of GeV^2.

$$\Pi_1(Q^2) = \frac{1}{(2\pi)^2}\left(-\frac{2m_\rho^2(2\pi)^2}{g_\rho^2(Q^2+m_\rho^2)} + \ln(1+\frac{s_0^*}{Q^2}\right),$$

where $\frac{g_\rho^2}{4\pi} = 2.3$ is residue of the ρ-meson, $m_\rho^2 = 0.6GeV^2$ is the ρ-meson mass squared, and $s_0^* = 1.5GeV^2$. Numerically in GeV units we have

$$\Pi_1(Q^2) = \frac{1}{(2\pi)^2}\left(-\frac{1.5}{Q^2+0.6} + \ln(1+\frac{1.5}{Q^2})\right)$$

$$\Pi_4(Q^2) = \frac{1}{2}\Pi_2(Q^2)$$

All these expressions for operators $\Pi_i(Q^2)$ were obtained in [7]. In (3.19 $s_0 = 4\,GeV^2$ is continuum threshold, which is numerically equal to the threshold of continuum in the sum rules for Δ-isobar mass. Such a choice for the value of the threshold follows from the assumption that there is one and the same value of threshold for different structures in the sum rules. Then, considering the sum rules for the electric charge (i.e. for the electric form factor $G_E(Q^2) = 1$, $Q^2 = 0$) it can be shown that they coincide with the mass sum rules except for the possibly different values of thresholds. But from Ward identity, we know that these sum rules should coincide exactly. So the value of the threshold in the sum rules is $s_0 \simeq 4GeV^2$ [2]. This assumption is based on the physical meaning of the quantity s_0, which as we know from the experience is determined mainly by the position of the next resonance in corresponding channel. And it would be very surprising if the resonance contribution to the electric form factor's channel strongly differs from the contribution to the magnetic form factor's channel.

In the limit $s_0 \to \infty$

$$\frac{1}{\pi^2}\int_0^\infty ds_1 \int_0^\infty ds2\rho_\Delta(s_1,s_2,Q^2)e^{-\frac{s_1+s_2}{2M^2}+z\frac{s_1-s_2}{M^2}} \tag{3.20}$$

$$= e_u M^6\left(2E_2(\frac{Q^2}{4M^2}(1-4z^2)) - 3E_3(fracQ^2 4M^2(1-4z^2)) + E_5(\frac{Q^2}{4M^2}(1-4z^2))\right)$$

Here $E_n(z) = \int_1^\infty x^{-n}e^{-zx}dx$.

Now, let us discuss the continuum contribution into the second term of the left side of (3.19). The double discontinuity of the corresponding diagram could not be calculated due to the reasons discussed in the Section 2 of this paper. We do not know the exact formula, analogous to the first term in (3.19), but can write an approximate expression, which becomes exact in the limit when $M^2 \to \infty$. Moreover, if one substitutes the first term in the left side of (3.19) by the analogous term

$$e_u M^6 (1 - e^{-s_0/M^2}(1 + \frac{s_0}{M^2} + \frac{s_0^2}{2M^4}))e^{Q^2/4M^2} \times \tag{3.21}$$

$$\int_{-1/2}^{+1/2} dz[2E_2(\frac{Q^2}{4M^2}(1-4z^2)) - 3E_3(\frac{Q^2}{4M^2}(1-4z^2)) + E_5(\frac{Q^2}{4M^2}(1-4z^2))]$$

then one sees that their differences is less then 10% in the region $Q^2 < 1GeV^2$. So, since we shall work in such a region of M^2, where the continuum contribution is less then $20-30\%$, then the uncertainties due to the second term in (3.19) will be less then a few percents.

To find $G_M(Q^2)$ we should study the following formula to kill nonsuppressed contribution of the nondiagonal transitions

$$G_M(Q^2) = 3e_u(1 - M^2\frac{\partial}{\partial M^2})\tilde{\lambda}^{-2}e^{m_\Delta^2/M^2}I(M^2, S_0, Q^2)$$

at any fixed Q^2.

The results obtained for the form factor are depicted in Fig.1. Notice that contribution of nondiagonal transitions is very small (about few percents) in the sum rule. In the region $0 \leq Q^2 \leq 0.8GeV^2$ the result obtained may be fitted by the following relation

$$G_M(Q^2) = \frac{6.16}{1 + Q^2/\mu^2}\left(\frac{e\hbar}{2m_p c}\right) \tag{3.22}$$

where $\mu^2 = 0.7\, GeV^2$.

The additive quark model prediction is $G_M(0) = 7.4\ \frac{e\hbar}{2m_p c}$.

The analogous sum rule could be also written in the case of Ω^--hyperon in the limit of $SU(3)$ symmetry after evident interchange $e_u \leftrightarrow e_s$. The accuracy of the results obtained is about $10-20\%$.

IV. CONCLUSIONS

In the present paper the QCD sum rules for the polarization operator in a variable external field are used to calculate the Δ-isobar magnetic form factor in the infra-red $0 \leq Q^2 < 1GeV^2$ region. Analogous sum rules can be used for calculation of other diagonal hadronic form factors in the infra-red region. Notice that this method does not work in the case of nondiagonal form factors because it is not possible to separate interesting contribution into correlator. There is only one way to do it: to sum all $(kx)^n$-terms in the expression for nonperturbative propagator (2.6). It was done only in the case $Q^2 = 0$ [12] using operator product expansion on a light cone and models for the photon wave functions.

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Figures

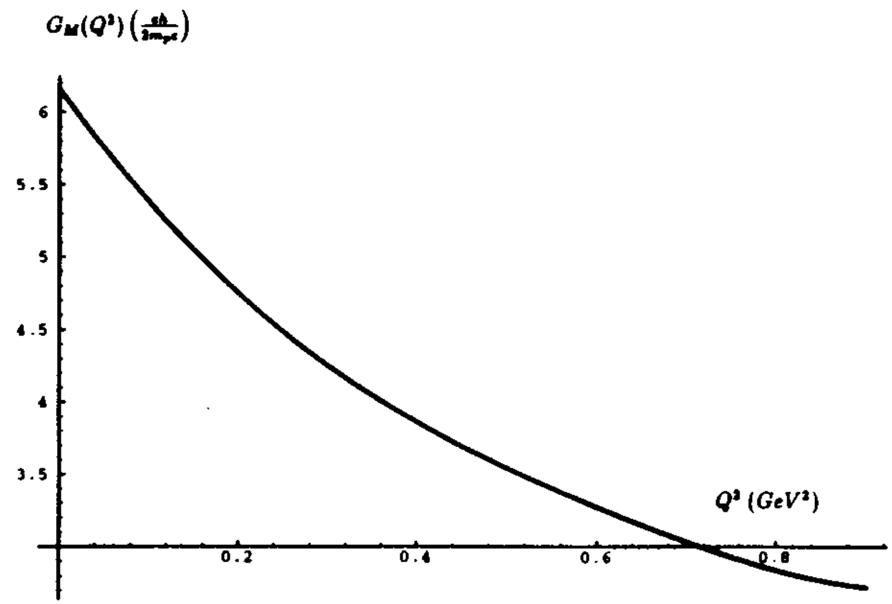

Fig.1

Delta-isobar magnetic form factor.